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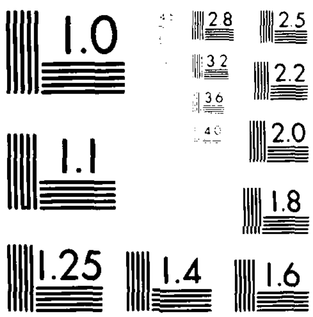

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ON THE NATURE OF BOUNDARY CONDITIONS FOR CRACK TIP STRESS

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ON THE NATURE OF BOUNDARY CONDITIONS FOR CRACK TIP STRESS*

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ABSTRACT

Our recent calculations on crack tip stresses [1,2,] were scrutinized by Atkinson [3], suggesting that our scheme of calculations is not uniform and the solution of the problem may not actually exist. Here we show that these difficulties arise because of the use of incorrect boundary conditions by Atkinson. Also, we give an exact solution of a problem which refutes the suggested inexistence.

1. INTRODUCTION

Recently, we gave solutions of some crack problems within the context of the theory of nonlocal elasticity. It was shown that [1,2] in nonlocal elasticity, crack tip singularity does not exist and the hoop stress reaches a maximum just outside crack, adjacent to it. By equating this maximum to the cohesive stress, we can find the ultimate stress at which crack begins to propagate. In this way, it was possible to establish a fracture criterion based on maximum stress hypothesis. Cohesive stress calculations and several other findings indicated excellent agreements withthe results of the atomic lattice dynamics and Griffith theory.

*The present work was supported by the Office of Naval Research.

Atkinson [3], by analyzing the one-dimensional model, suggested that our scheme of calculation may have a *non-uniform* character, and the problem posed and solved numerically may not actually possess a solution. Here, we show that this is not the case. Had he used the correct boundary condition in analyzing the hoop stress, these questions do not arise. Basically, Atkinson's difficulty is due to the use of the infinite range $(-\infty, \infty)$ in calculating the integral for the stress within the crack surface and using nonlocal kernels without support. More precisely, the influence of strains outside crack line must be excluded in calculating the stress within the crack. Otherwise, an incompatibility will exist in the boundary conditions.

We also give the exact solution of a problem which indicates that the solution of the one-dimensional crack with a given load distribution exists.

2. NON-LOCAL ELASTICITY

Basic equations of linear, isotropic, nonlocal elasticity [4,5] with vanishing inertia and body forces consist of equations of equilibrium.

$$(2.1) \qquad t_{k\ell,k} = 0 \qquad \text{in } V$$

and constitutive equations

$$(2.2) \qquad t_{k\ell} = \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\, \sigma_{k\ell}(\underset{\sim}{x}')\, dv(\underset{\sim}{x}')$$

where

$$\sigma_{k\ell} = \lambda\, u_{r,r}\, \delta_{k\ell} + \mu(u_{k,\ell} + u_{\ell,k}) \tag{2.3}$$

Here, $t_{k\ell}$ is the stress tensor and u_k is the displacement vector, referred to rectangular coordinates x_k, λ, and μ are Lamé constants and $\delta_{k\ell}$ is the Kronecker delta. The nonlocal modulus $\alpha(|\underset{\sim}{x}|)$ is subject to

$$\int_V \alpha(|\underset{\sim}{x}|)\, dv = 1 \tag{2.4}$$

Since classical elasticity limit is desired, $\alpha(|\underset{\sim}{x}|)$ must be a Dirac δ-sequence so that in the limit

$$\alpha = \delta(\underset{\sim}{x}) \tag{2.5}$$

nonlocal elasticity reverts to classical elasticity.

Equations (2.1) and (2.2), when combined, gives

$$\int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\, \sigma_{k\ell,k}(\underset{\sim}{x}')\, dv(\underset{\sim}{x}') - \int_{\partial V} \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\, \sigma_{k\ell}(\underset{\sim}{x}')\, da_k(\underset{\sim}{x}') = 0 \tag{2.6}$$

When (2.3) is substituted into (2.6), we obtain three integro-partial differential equations which must be solved under a set of boundary conditions to determine $u_k(x)$.

3. ONE-DIMENSIONAL MODEL

The one-dimensional model discussed in [1] is defined by

$$\gamma^2\, v_{xx} + v_{yy} = 0 \quad , \qquad y > 0 \tag{3.1}$$

subject to boundary conditions

$$(3.2) \qquad \begin{aligned} t_{yy}(x,0) &= -t_0(x), \qquad & |x| < \ell \\ v(x,0) &= 0 \qquad , & |x| > \ell \\ v \to 0 \quad &\text{as} \quad y \to \infty \end{aligned}$$

Here, $\gamma^2 = \mu/(\lambda+2\mu)$, v is the *y*-component of the displacement field, t_{yy} is the normal stress given by

$$(3.3) \qquad t_{yy}(x,y) = \int_{-\infty}^{\infty} \alpha(|x'-x|)\, \sigma_{yy}(x',y)\, dx' \quad , \qquad y > 0$$

where

$$(3.4) \qquad \sigma_{yy} = (\lambda + 2\mu) \frac{\partial v(x,y)}{\partial y} \quad .$$

It is important to remember that (3.3) is valid for $y > 0$. Since at $y = 0$ there is a crack located at $|x| < \ell$, $y = 0$, the influence of strains in $|x| > \ell$ on the stress in $|x| < \ell$ are forbidden so that the boundary condition $(3.2)_1$ reads

$$(3.5) \qquad (\lambda+2\mu) \int_{-\ell}^{\ell} \alpha(|x'-x|) \frac{\partial v(x',0)}{\partial y}\, dx' = -t_0(x) \; , \qquad |x| < \ell$$

As we stated in [1], the one-dimensional model cannot be derived rationally from the field equations so that we place no faith in this model. It was treated for computational reasons in support of the two-dimensional

solution discussed there. Since it was considered by Atkinson at length to discuss the questions of non-uniformity in calculations and possible failure of the existence of solution, we consider these questions in the context of correct mathematical and physical considerations.

Several non-local kernels are possible. We mention three:

$$\begin{aligned} \alpha(|x|) &= \frac{1}{a}\left(1 - \frac{|x|}{a}\right), \qquad |x| < a \\ &= 0 \qquad\qquad\quad\;, \qquad |x| > a \end{aligned} \tag{3.6}$$

$$\alpha(|x|) = \frac{\beta}{a\sqrt{\pi}} \exp\left[-\left(\frac{\beta}{a}\right)^2 x^2\right] \tag{3.7}$$

$$\alpha(|x|) = \frac{\beta}{2} e^{-\beta|x|} \tag{3.8}$$

Note that all these kernels are Dirac delta sequence, but only (3.6) has a *finite support*.

To clarify the physics of the problem, let us examine the nature of the boundary conditions. In Fig. 1 there is shown a perfect lattice with a crack of length 2ℓ. It is clear that an arbitrary load can only be applied in the region

$$|x| < \ell - a \tag{3.9}$$

where a is the atomic distance. Therefore, the support of the kernel $\alpha(|x|)$ must be finite as in (3.6) and then the range of the applied stress cannot be all the way to $x = \pm \ell$. If we pass to the nonlocal continuum limit this implies that the kernel $\alpha(|x|)$ must be cut off* at $x = \pm \ell$ or else an

*In fact, the absence of this cut-off implies that there is no crack in the perfect lattice unless we introduce an edge dislocation (finite jump in v for $|x| < \ell$) together with the applied load.

inhomogeneous kernel $\alpha(x,x')$ must be used within regions

$$(3.10) \qquad -\ell < x < -\ell + a \quad , \qquad \ell - a < x < \ell$$

when (3.6) is employed. Kernels (3.7) and (3.8) do not possess finite support. Therefore, they must be cut off for $|x| \geq \ell$.

From the mathematical point of view, it is clear that if $t_{yy}(x,0)$ is calculated by

$$(3.11) \quad t_{yy}(x,0) = (\lambda+2\mu) \int_{-\infty}^{\infty} \alpha(|x'-x|) \frac{\partial v(x',0)}{\partial y} dx' = - t_0(x) \quad ,$$

in general, boundary conditions $(3.2)_1$ and $(3.2)_2$ will be *incompatible*. This is because $v(x,0) = 0$ is already specified for $|x| \geq \ell$ and that prescription of $t_{yy}(x,0)$ for $|x| < \ell$ imposes conditions on $\partial v/\partial y$ within the common region to the support of $\alpha(|x|)$ and $|x| > \ell$. In the case of the kernel (3.6), this region is given by $\ell \leq |x| \leq \ell + a$, for kernels (3.7) and (3.8), it is the entire region $|x| > \ell$.

Atkinson investigates the behavior of stress t_{yy} by using (3.11) in the crack region and finds that the boundary condition is satisfied uniformly for $|x| < \ell - a$, as $a \to 0$, but in the regions $\ell - a < |x| < \ell$ (one atomic distances near the crack tips!), t_{yy} does not approach to its constant value uniformly as $a \to 0$ (in the case of kernels (3.7) and (3.8) as $\beta \to 0$) .*

In Section 4 below, we will show that: <u>If the correct boundary conditions (3.5) for t_{yy} is used, then the boundary condition on t_{yy} is satisfied uniformly.</u>

*Perhaps what misled him to these conclusions is a missing statement from our work, that the Fourier transform of (3.11) is only an approximation to that of the exact boundary condition (3.5).

4. EVALUATION OF THE STRESS

The Fourier representation of $v(x,y)$ satisfying (3.1) and $(3.2)_3$ is given by

$$(4.1) \qquad v(x,y) = (2/\pi)^{\frac{1}{2}} \int_0^\infty A(k)\, e^{-\gamma k y} \cos(kx)\, dk$$

The correct boundary conditions $(3.2)_{1,2}$ read

$$(4.2) \qquad \begin{aligned} t_{yy}(x,0) &= -(2/\pi)^{\frac{1}{2}} (\lambda+2\mu)\gamma \int_{-\ell}^{\ell} \alpha(|x'-x|)\, dx' \int_0^\infty k\, A(k) \cos(kx) dk \\ &= -t_0(x), \qquad 0 < x < \ell \\ \int_0^\infty A(k) \cos(kx)\, ak &= 0\,, \qquad x > \ell \end{aligned}$$

In [1] for $A(k)$ we employed

$$(4.3) \qquad A(k) \simeq A_c(k) = \ell\, t_0[2\mu(\lambda+2\mu)/\pi]^{-\frac{1}{2}} J_1(k\ell)/k$$

which is the exact solution of the same problem in classical elasticity. Clearly (4.3) satisfies $(4.2)_2$ identically and for $(4.2)_1$ gives

$$(4.4) \qquad t_{yy}(x,0)/t_0 = -\int_{-\ell}^{\ell} \alpha(|x'-x|)\, dx' \qquad 0 < x < \ell$$

Evaluating this integral with $\alpha(|x|)$ given by (3.8), we obtain:

$$(4.5) \qquad t_{yy}/t_0 = -1 + e^{-\beta \ell} \cosh \beta x \quad , \qquad |x| < \ell$$

Using the reasoning of Atkinson [3], we set $\beta x = X$, then

$$(4.6) \qquad P_c(x,\beta) = (t_{yy} + t_0)/t_0 = e^{-\beta \ell} \cosh X \ .$$

From this it is clear that as $\beta \to \infty$, for $x < \ell$, $P_c(x,\beta)$ tends to zero uniformly in β.

At the crack tips, $x = \pm \ell$, which is not included in the region of validity of (4.5), we have

$$(4.7) \qquad P_c(\ell,\beta) \to \frac{1}{2} \qquad \text{as} \qquad \beta \to \infty$$

A similar situation is valid for the kernel (3.7):

$$(4.8) \qquad t_{yy}/t_0 = -\frac{1}{2}\Phi[\beta(\ell + x)/a] - \frac{1}{2}\Phi[\beta(\ell - x)/a]$$

where $\Phi(z)$ is the Fresnel integral defined by

$$(4.9) \qquad \Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2}\, dx$$

For the kernel (3.6), we find

$$(4.10) \qquad \begin{aligned} t_{yy}/t_0 &= -1 \ , & 0 < x \le \ell - a \\ t_{yy}/t_0 &= -\frac{1}{2} - \frac{\ell - x}{a}\left(1 - \frac{\ell - x}{2a}\right) \ , & \ell - a < x < \ell \end{aligned}$$

which shows that the boundary conditions on stress is satisfied all the way up to one atomic distance away from the crack tip. Beyond this, t_{yy} goes from $-t_0$ at $|x| = \ell - a$ to $-t_0/2$ at $|x| = \ell$.*

The stress field outside the crack is given by

$$t_{yy}/t_0 = \ell^2 \int_{\ell}^{\infty} \frac{\alpha(|x'-x|)\, dx'}{(x'^2-\ell^2)^{\frac{1}{2}}\,[x' + (x'^2-\ell^2)^{\frac{1}{2}}]} \,, \quad x > \ell \tag{4.11}$$

If we let

$$x' = \ell \cosh y \tag{4.12}$$

This integral becomes

$$t_{yy}/t_0 = \ell \int_0^{\infty} e^{-y}\, \alpha(|\ell \cosh y - x|)\, dy \tag{4.13}$$

which is bounded for all $x > \ell$.

If we use the kernel (3.8) and carry out the integration in (4.13) for $x = \ell$, we obtain

$$t_{yy}/t_0 = \frac{1}{2}\beta\ell\, e^{\beta\ell} K_1(\beta\ell) - \frac{1}{2} \tag{4.14}$$

where $K_1(x)$ is the Bessel's function. For large $\beta\ell$, this gives

$$C = (\beta\ell)^{-\frac{1}{2}} P(\ell) \simeq \frac{1}{2}(\pi/2)^{\frac{1}{2}} \simeq 0.8 \tag{4.15}$$

---

*Note that in this region, $\alpha(|x|)$ is not normalized to unity. If we normalize it by dividing it with its area (which is 1/2 at $|x| = \ell$), we would obtain $t_{yy} = -t_0$ at $|x| = \ell$.

where

$$(4.16) \qquad P(\ell) \equiv [t_{yy}(\ell,0)/t_0] + 1 \ .$$

For the kernel (3.6), the integration of (4.13) gives

$$(4.17) \qquad t_{yy}(x,0)/t_0 = \beta f(x) \ ,$$

where

$$(4.18) \qquad f(x) = 1 - \frac{x}{a} + \frac{\ell}{a}(\frac{x}{\ell})^2 - 2\frac{x}{a}[(x/\ell)^2 - 1]^{\frac{1}{2}} - \frac{2x}{\ell} - \frac{a}{\ell}$$

$$+ (1 + \frac{x}{a})[(\frac{x+a}{\ell})^2 - 1]^{\frac{1}{2}} - \frac{\ell}{a}(x/\ell)^2 + \frac{\ell}{2a} + \frac{x}{a}[(x/\ell)^2 - 1]^{\frac{1}{2}}$$

$$+ \frac{\ell}{a}\,\mathrm{Arch}(x/\ell) - \frac{\ell}{2a}\,\mathrm{Arch}\,(\frac{x+a}{\ell}) + \frac{\ell}{2a}(\frac{x+a}{\ell})^2$$

$$- \frac{\ell}{2a}(\frac{x+a}{\ell})[(\frac{x+a}{\ell})^2 - 1]^{\frac{1}{2}} \qquad \ell \le x < \ell + a$$

For $x = \ell$, we obtain the hoop stress at the crack tip.

$$(4.19) \qquad t_{yy}(\ell,0)/t_0 = -\frac{1}{2} + (2\ell/a)^{\frac{1}{2}}\,\frac{1+(\ell/a)}{2}\,(1 + \frac{a}{2\ell})^{\frac{1}{2}} - \frac{1}{2}(\ell/a)^2\,\mathrm{Arch}(1 + \frac{a}{\ell})$$

exactly. For large ℓ/a, this gives

$$(4.20) \qquad t_{yy}(\ell,0)/t_0 \simeq -\frac{1}{2} + \frac{2}{3}(2\ell/a)^{\frac{1}{2}}$$

Consequently,

$$(4.21) \qquad C = (a/2\ell)^{\frac{1}{2}}\,P(\ell) = \frac{2}{3} \ .$$

These results compare well with our previous computer results given in [1].

We notice that the hoop stress experiences a jump discontinuity across crack tips (Figure 2). This is as expected, since the region of influence of strains on stresses within and outside the crack were terminated at crack tips. By employing an inhomogeneous kernel $\alpha(x,x')$ that vanishes at crack tips, continuity in the stress field can be maintained across the tips. However, in this case, the mathematical problem becomes much more difficult to tackle.

5. QUESTION OF EXISTENCE

In order to understand the correct behavior of the hoop stress at the crack tips, $x = \pm \ell$, we note that for $\alpha(|x|)$ given by (3.8) by differentiation from (3.5), we get

$$(5.1) \qquad \sigma_{yy} = (\lambda + 2\mu)\frac{\partial v}{\partial y} = t_{yy} - \frac{1}{\beta^2}\frac{\partial^2 t_{yy}}{\partial x^2}$$

which is valid also for (3.3). At first sight, it appears that $t_{yy} = -t_0 =$ const. in $|x| \leq \ell$ gives $\sigma_{yy} = -t_0 =$ const., so that the classical elasticity solution for v is in fact the solution of the nonlocal problem. The fact that this is not the case can be seen if we set $\sigma_{yy} = -t_0 =$ const., in (3.5) we get (4.5). This is because

$$(5.2) \qquad t_{yy} = -t_0 + A e^{-\beta x} + B e^{\beta x}$$

where A and B are constants, also gives $\sigma_{yy} = -t_0 =$ const.. Clearly, Eq. (5.1) is not equivalent to Eq. (3.5) unless we adjoin to it two appropriate boundary conditions at $x = \pm \ell$. In the case of (3.3), these conditions are $t_{yy} = 0$ for $x = \pm\infty$. In the case of (3.5), the region $|x| \geq \ell$ is forbidden for the calculation of t_{yy} in the crack region $|x| < \ell$. Similarly, for the evaluation of the stress, t_{yy} outside the crack, the region $|x| < \ell$ is forbidden.

In Section 3 of [3], Atkinson employs an integral equation (his Eq. (3.5)) to demonstrate that the solution of the problem (using still another kernel) may not exist. Under the correct boundary conditions (4.2), (his

Eq. (3.5)) will have to be modified to eliminate the effect of strains in $|x| \geq \ell$ on the stress in $|x| < \ell$. On the basis of the correct integral equation that replaces his Eq. (3.5), it can be seen that all of his arguments regarding inexistence are no longer valid.

For the stress boundary condition (4.5), in fact the boundary conditions are satisfied exactly. $v(x,y)$ is identical to the classical solution, the hoop stress t_{yy} for $x > \ell$ is given by (4.13) which has no singularity.

We have therefore found an *exact* solution of the problem. This solution also uniformly approximates the case of constant loading for large $\beta\ell$, to a very high degree of accuracy in $|x| < \ell$. For example, for steel $a \approx 2.48 \times 10^{-8}$ cm. Even for a microcrack of length $2\ell = 6 \times 10^{-4}$ cm. we have $\beta\ell \approx 10^4$ so that the error in t_{yy} is

$$(5.3) \qquad P_c \approx \frac{1}{2} \exp[-10^4(1 - x/\ell)]$$

If we accept an error $P_c < 10^{-3}$, t_{yy} will be nearly constant all along the crack up to the point

$$(5.4) \qquad x/\ell = 1 - 3 \times 10^{-4} \ \ell n(20)$$

for kernels having finite support such as (3.6). The error in the satisfaction of the boundary condition on stress begins only in intervals within one atomic distances from the crack tips.

6. CONCLUSION

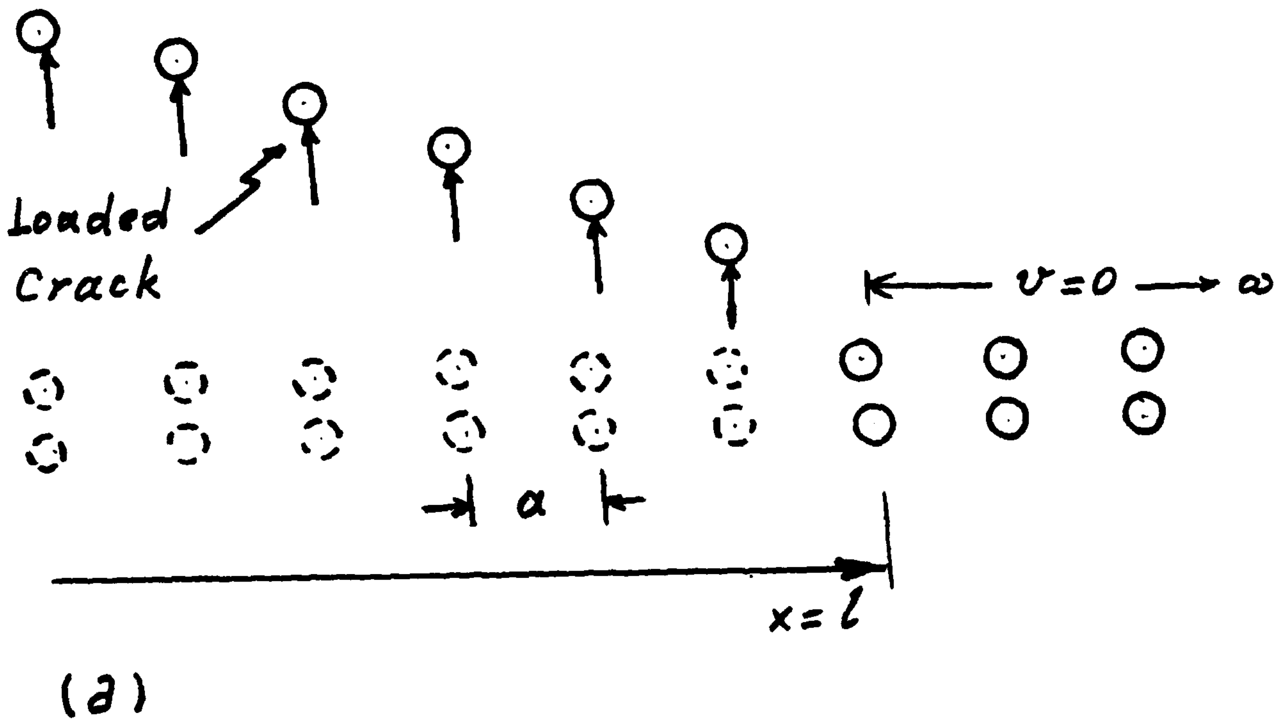
The non-uniformity in the satisfaction of the stress boundary conditions and possible failure of existence of solution noted by Atkinson [3] is due to his use of the infinite range for the integral for analyzing the stress near crack tips. In nonlocal elasticity, the stress at a point is influenced by strains at all other points within the support of the kernel. If the kernel has no support, this brings the influence of strains at points outside the crack to the stress specified within the crack. If the support of the kernel is finite, then only the set of points within the intersection of the support and the complement of the set of points in the crack region influences the stress. This is not compatible with the specification of the displacement field in this common region which is outside the crack. To restore the compatibility to the mixed boundary conditions, we must exclude the influence of strains on the stress in the common regions, to the support of the kernel and the outside crack. This can be done either by limiting the range of integration to the crack length or by using an inhomogeneous kernel (in regions near the crack tips) whose support is confined to the crack line, varying with distance from the crack tip. This latter approach, while correct mathematically, is cumbersome and presents major analytical difficulties. Nevertheless, it indicates that a solid with surfaces and cuts is an inhomogeneous body and the use of homogeneous kernels represents an approximation.

ACKNOWLEDGEMENT

Dr. Atkinson was kind enough to send me his manuscript before its publication. Unfortunately, I could not devote sufficient time to his work to supply him with a detailed answer earlier. I acknowledge, with thanks, various discussions I have had with my students, Nasit Ari, A. Suresh and V. Chen and with Professors Speziale and Srivastav, on this and other related questions.

Post Script

After this work was completed, it was brought to my attention that Prof. Atkinson published another paper on the same questions related to the plane shear, plane stress and antiplane shear problems (Arch. Mech., 32, 4, pp. 597-614, Warszawa, 1980). In this paper, he uses the same type of wrong boundary conditions discussed above. Hence the present conclusions are valid for this paper as well.



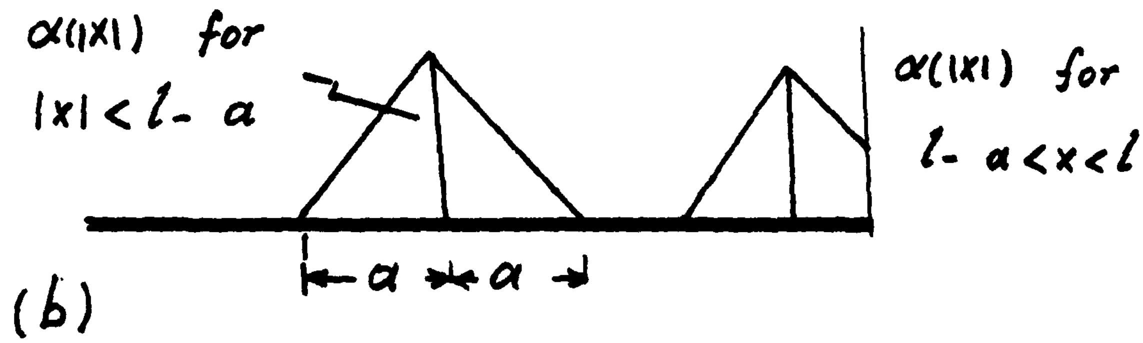


Figure 1: a) Perfect lattice with loaded crack.
b) Influence kernel $\alpha(|x|)$ for nonlocal continuum.

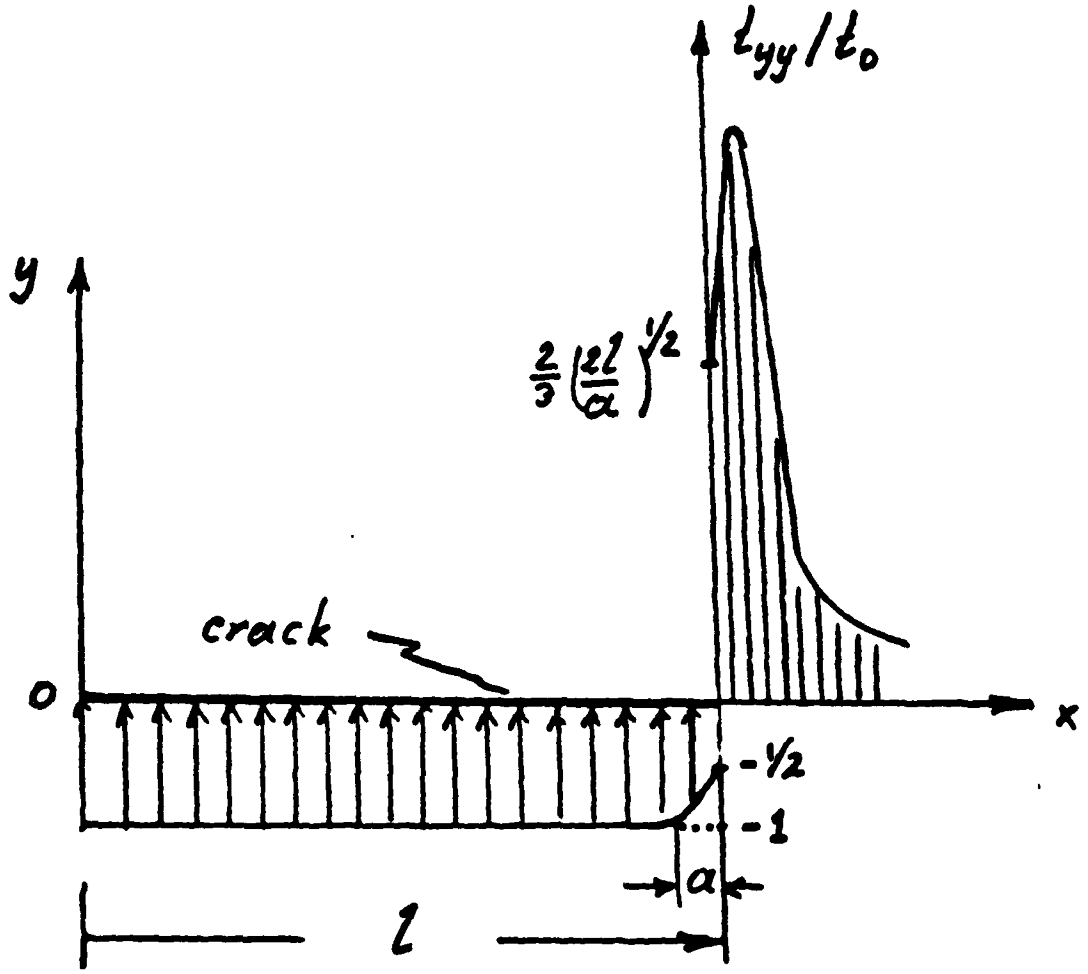


FIGURE 2

Stress Distribution along crack line.

l = half crack length

a = Atomic distance.

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Air Force Office of Scientific Research
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Department of the Air Force
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